

This segment from a news program details the training regiment for Dana Carellas, a 15-year old figure skater. In a number of scenes, she performs a spin; as she comes out of the spin, she extends her arms out to the side to slow down. This is a typical problem utilizing the conservation of angular momentum in which the object's moment of inertia changes so the angular velocity changes as well.

First, consider how her moment of inertia changes. Initially, Dana can be best represented by a spinning cylinder. Assuming her mass to be 50 kg and her "radius" to be 0.2 m, her moment of inertia is:

$$I_{cylinder} = \frac{1}{2}MR^2 = \frac{1}{2}(50 \text{ kg})(0.2\text{m})^2 = 1 \text{ kg m}^2$$

Then, when Dana extends her arms out, her body is best approximated by a solid cylinder rotating about its central axis (her body) and a thin rod about an axis through center perpendicular to the length. Assuming her armspan (from fingertip to fingertip) to be 1.5 m and the mass of her arms to be 8 kg, her new moment of inertia is:

$$I_{cylinder} + I_{thin \text{ rod}} = \frac{1}{2}MR^2 + \frac{1}{12}ML^2 = \frac{1}{2}(42 \text{ kg})(0.2\text{m})^2 + \frac{1}{12}(8 \text{ kg})(1.5 \text{ m})^2 = 2 \text{ kg m}^2$$

Now, consider that Dana is spinning at 2 revolutions per second when her arms are pulled in. What is her new angular speed when she extends her arms?

The law of conservation of angular momentum states:

$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

$$(1 \text{ kg m}^2) \left(2 \frac{\text{rev}}{\text{s}}\right) = (2 \text{ kg m}^2) \omega_2$$

$$\omega_2 = 0.5 \frac{\text{rev}}{\text{s}}$$

So, Dana can decrease her angular speed by a factor of 2. What can she do to increase her angular speed? What are some other things she can do with her body to alter her speed?